Pascal and the Risk Assessment Code (RAC) Matrix

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Abstract

In 1662, a basic concept for risk assessment was first published: "*Risk should be proportional to both likelihood and consequence.*" This concept underpins the Risk Assessment Code (RAC) that is in use today. The present paper examines the historical precepts more closely and draws conclusions about where we are today, and where we could be.

Introduction - Highlights from History

In recent years, excellent histories have been written about the development of the theories and mathematics supporting risk analyses (refs. 1, 2, 3). This paper only hits a few of the highlights. Figure 1 depicts the major developments in theory and principles associated with the assessment of risk. Two timelines are shown; the upper one spans 8500 years of history up until 1500 AD, and the lower timeline depicts the last 500 years.

Major developments in many numerically intense areas, such as geometry and astronomy, can be traced back more than 6,000 years. In contrast, the major developments in probability theory that underpin risk assessment were not developed until the last 500 years. The reason is simple: the most basic tool to allow that development was not available previously - it is the *Arabic numbering system* that first facilitated computation and communication. Probability theory deals with the number space between zero and one, and Roman numerals - which were the scale of choice in Europe until about 1500 - have no zero. Once the Arabic numbering system took hold and mathematicians began to use it, new theory was postulated and developed at a relatively rapid pace.

It is interesting to note that it took about 500 years for the "new" numbering system to become widely accepted. This time span seems disappointingly long in the context of today's

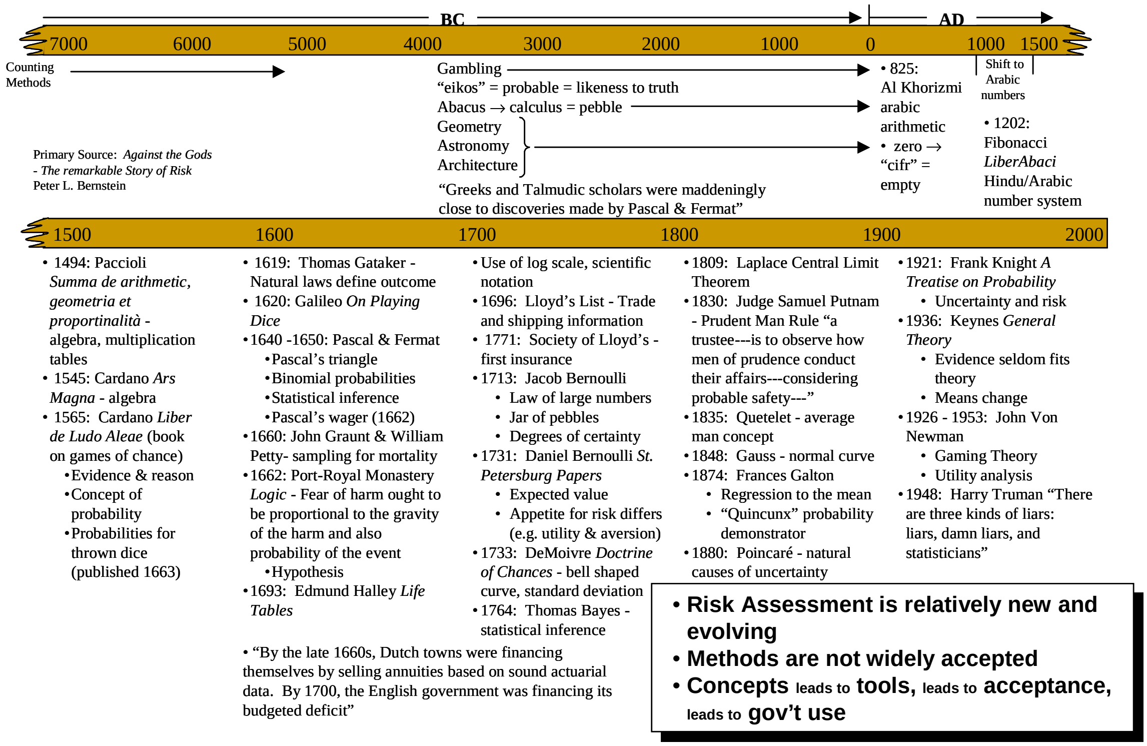
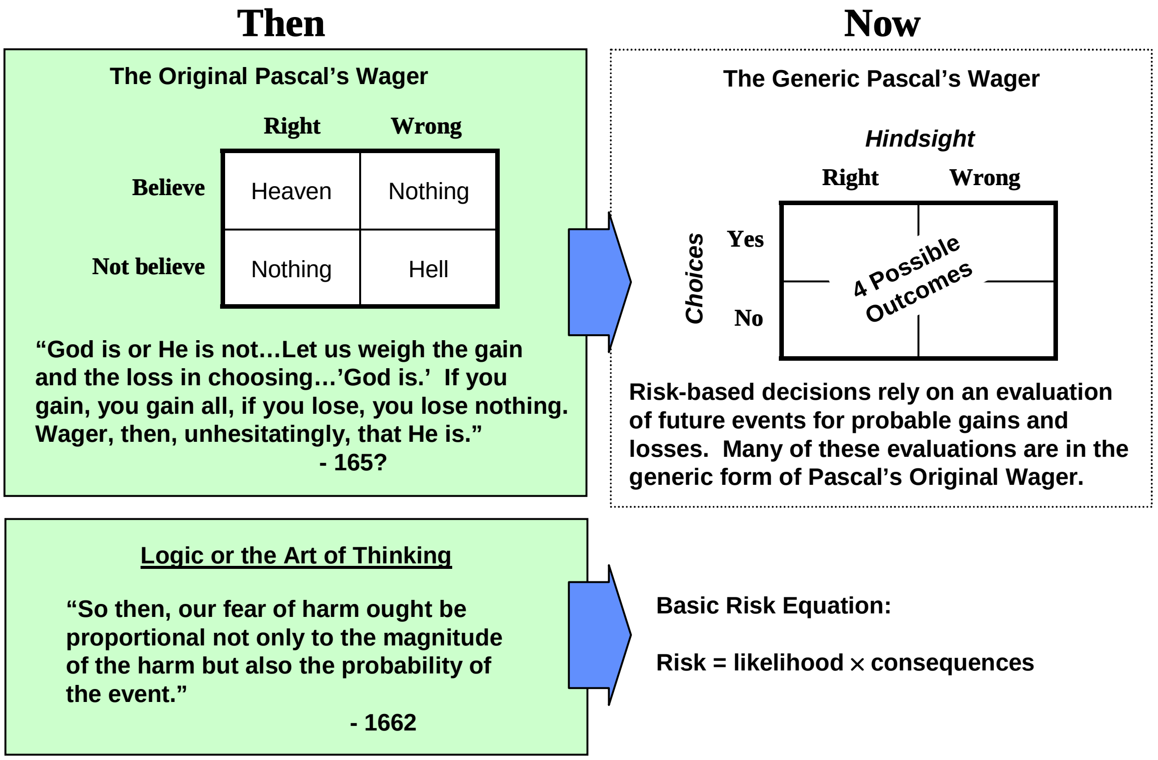

BC
AD
7000
6000
5000
4000
3000
2000
1000
0
1000
1500
Counting Methods
Gambling
"eikos" = probable = likeness to truth
Abacus → calculus = pebble
Geometry
Astronomy
Architecture
"Greeks and Talmudic scholars were maddeningly close to discoveries made by Pascal & Fermat"
• 825: Al Khorizmi arabic arithmetic
• zero → "cifr" = empty
Shift to Arabic numbers
• 1202: Fibonacci *LiberAbaci* Hindu/Arabic number system
Primary Source: *Against the Gods - The remarkable Story of Risk* Peter L. Bernstein
1500
1600
1700
1800
1900
2000
• 1494: Paccioli *Summa de arithmetic, geometria et proportinalità* - algebra, multiplication tables
• 1545: Cardano *Ars Magna* - algebra
• 1565: Cardano *Liber de Ludo Aleae* (book on games of chance)
• Evidence & reason
• Concept of probability
• Probabilities for thrown dice (published 1663)
• 1619: Thomas Gataker - Natural laws define outcome
• 1620: Galileo *On Playing Dice*
• 1640 -1650: Pascal & Fermat
• Pascal's triangle
• Binomial probabilities
• Statistical inference
• Pascal's wager (1662)
• 1660: John Graunt & William Petty- sampling for mortality
• 1662: Port-Royal Monastery *Logic* - Fear of harm ought to be proportional to the gravity of the harm and also probability of the event
• Hypothesis
• 1693: Edmund Halley *Life Tables*
• "By the late 1660s, Dutch towns were financing themselves by selling annuities based on sound actuarial data. By 1700, the English government was financing its budgeted deficit"
• Use of log scale, scientific notation
• 1696: Lloyd's List - Trade and shipping information
• 1771: Society of Lloyd's - first insurance
• 1713: Jacob Bernoulli
• Law of large numbers
• Jar of pebbles
• Degrees of certainty
• 1731: Daniel Bernoulli *St. Petersburg Papers*
• Expected value
• Appetite for risk differs (e.g. utility & aversion)
• 1733: DeMoivre *Doctrine of Chances* - bell shaped curve, standard deviation
• 1764: Thomas Bayes - statistical inference
• 1809: Laplace Central Limit Theorem
• 1830: Judge Samuel Putnam - Prudent Man Rule "a trustee---is to observe how men of prudence conduct their affairs---considering probable safety---"
• 1835: Quetelet - average man concept
• 1848: Gauss - normal curve
• 1874: Frances Galton
• Regression to the mean
• "Quincunx" probability demonstrator
• 1880: Poincaré - natural causes of uncertainty
• 1921: Frank Knight *A Treatise on Probability*
• Uncertainty and risk
• 1936: Keynes *General Theory*
• Evidence seldom fits theory
• Means change
• 1926 - 1953: John Von Newman
• Gaming Theory
• Utility analysis
• 1948: Harry Truman "There are three kinds of liars: liars, damn liars, and statisticians"
• Risk Assessment is relatively new and evolving
• Methods are not widely accepted
• Concepts leads to tools, leads to acceptance, leads to gov't use

Figure 1 - Quantitative Risk Assessment (QRA) Timeline

information age. Why should it take so long to adopt a concept that is obviously better?

Today’s use of risk assessment is supported by many significant developments. These include: (1) the concept of probability first documented in 1565, (2) the concept and use of the logarithmic scale and scientific notation in the 1700's, (3) Lloyds List, which later became Lloyds of London, beginning in 1696, (4) the concept of expected value documented in 1731, (5) the legal concept of the “prudent man” established in 1830, (6) the normal curve in 1848, and many others. It is not the purpose of this paper to review all these developments. However, for the purposes of comparison and contrast this paper focuses more sharply on the period of the mid 1600's and the contributions made by the mathematician Blaise Pascal.

Pascal has been recognized as one of history's foremost contributors to the field of mathematics (ref. 4)**.** His contributions in original thinking are many and range across science, mathematics, and religion. One of his contributions, “Pascal's Wager,” is more widely known for its philosophical and religious connotations than the underlying logic. However, as shown in figure 2, the form of a decision matrix used in the wager is widely applicable to problems in decision theory. Government agencies, safety organizations, and individuals alike use an inherently simple process of examining alternative potential outcomes and making decisions as to the acceptability of the risk. The generic form of Pascal's Wager is often the method used. Within a few years after the publication of Pascal's Wager, his logic became more sharply focused on the application of risk to safety. More of Pascal's work was published and widely reprinted in a book, compiled by Arnauld, that has become a classic: “Logic or the Art of Thinking” (ref. 1). In this book the first clear statement of the theory underpinning our risk matrix appears. It is compelling inasmuch as it was written over 300 years ago and points the way ahead for risk analysis in general. To show the context, a more lengthy quote is included here as figure 3.



The Original Pascal’s Wager

	Right	Wrong
Believe	Heaven	Nothing
Not believe	Nothing	Hell

“God is or He is not…Let us weigh the gain and the loss in choosing…’God is.’ If you gain, you gain all, if you lose, you lose nothing. Wager, then, unhesitatingly, that He is.”
- 165?

The Generic Pascal’s Wager

Risk-based decisions rely on an evaluation of future events for probable gains and losses. Many of these evaluations are in the generic form of Pascal’s Original Wager.

Logic or the Art of Thinking

“So then, our fear of harm ought be proportional not only to the magnitude of the harm but also the probability of the event.”
- 1662

Basic Risk Equation:

Risk = likelihood × consequences

Figure 2 – Blaise Pascal, “The Father of Risk-Based Decisions”

Chapter 16. Judgments we make concerning future accidents.

“These rules, [referring to earlier chapters] *which are helpful for judging about past events, can be easily applied to future events…*
“Many people, for example, are exceedingly frightened when they hear thunder. If thunder makes them think of God and death and happiness, we would not think about it too much. But if it is only the danger of dying by lightning that causes them this unusual apprehension, it is easy to show that this is unreasonable. For out of two million people, at most there is one who dies this way. We could even say that there is hardly a violent death that is less common. So, then, our fear of some harm ought to be proportional not only to the magnitude of the harm, but also to the probability of the event. Just as there is hardly any kind of death more rare than being struck by lightning, there is also hardly any that ought to cause less fear.”

Figure 3 - Logic or the Art of Thinking, Quote from Chapter 16

From this line of thinking, the basic risk equation in widespread use is derived directly:

Risk = Likelihood x Consequences

Moreover, there is little doubt that Pascal, the mathematician, postulated the words with their more concise and mathematical definition in mind, because in the same volume we find his rules for use by analysts (ref. 1). It is at first surprising to see how applicable those rules are to the quantitative risk analysis conducted today.

More Recent History

The application of risk assessment to major Government sponsored programs during our lifetime spans a wide variety. A sampling is illustrated in figure 4.

There have been some notable successes. For example, the National Ranges have applied a highly developed form of risk assessment to the approval process for launching from national ranges (ref. 5). The near perfect safety record in protecting the public from an activity with inherent potential hazards is probably unparalled when compared to any other major technology development.

Risk Assessment in the area of Explosives Safety has grown significantly in the last few years within the US (ref. 6). However, it has been successfully applied in several European Countries for decades. The Swiss are generally credited for leading the way.

Other highly visible events have caused the discipline of risk assessment to become the subject of skepticism. The Challenger Space Shuttle disaster, Three Mile Island, and others provide examples.

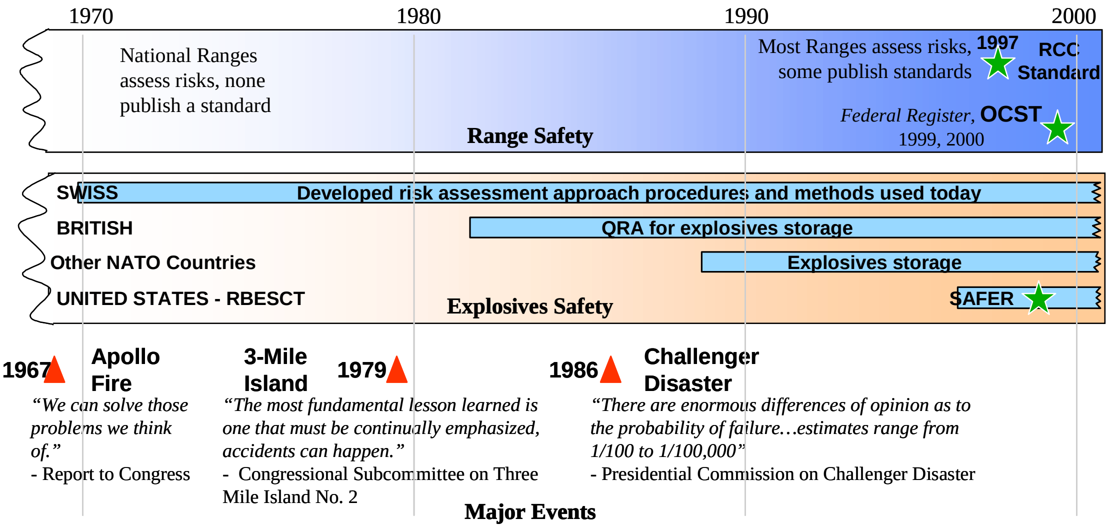

1970
1980
1990
2000
National Ranges assess risks, none publish a standard
Range Safety
Most Ranges assess risks, some publish standards
1997
RCC Standard
Federal Register, 1999, 2000
OCST
SWISS
Developed risk assessment approach procedures and methods used today
BRITISH
QRA for explosives storage
Other NATO Countries
Explosives storage
UNITED STATES - RBESCT
SAFER
Explosives Safety
1967
Apollo Fire
“We can solve those problems we think of.”
- Report to Congress
3-Mile Island
1979
“The most fundamental lesson learned is one that must be continually emphasized, accidents can happen.”
- Congressional Subcommittee on Three Mile Island No. 2
1986
Challenger Disaster
“There are enormous differences of opinion as to the probability of failure…estimates range from 1/100 to 1/100,000”
- Presidential Commission on Challenger Disaster
Major Events

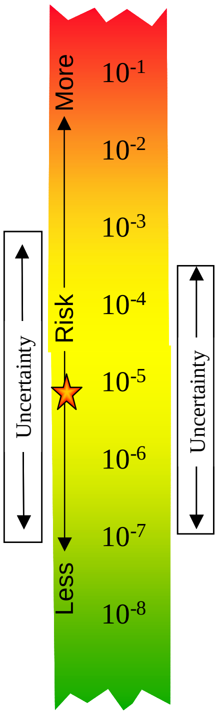
Figure 4 – Some Recent History

Approach 1. Specify safety criteria.

Major Issue: How safe is safe enough?

Determination involves:
- **Social science**
- **Legal considerations**

Approach 3. Combining Approach 1 and 2 provides the highest assurance of fair and impartial governance. Also: better credibility, lower cost, time saver, and less uncertainty



Approach 2. Specify quantitative risk assessment.

Major Issue: How to calculate?

Determination involves:
- **Physical Science**
- **Technical assumptions**
- **Technical approaches**
- **Biases (worst case ←→ self-interest)**

Figure 5 – Governing Safety Using Quantitative Risk Assessment

Growing Government use

Requirements for risk assessments and associated criteria appear in a growing number of regulatory documents in use worldwide today. It is interesting to examine the approaches used by governing bodies responsible for assuring safety (figure 5). The approaches fall into one of three generic forms:

- The first general case is to specify quantitative criteria. Here the criteria form the basis of acceptable risk and the analyst may have freedom to conduct the assessment using a wide variety of methods.
- The second includes the requirement to do a risk assessment. For this case, there are no stated criteria and there is often no analysis methodology specified. Currently, the Risk Assessment Code (RAC) matrix is predominately used in this manner.
- Both of these methods result in wide latitude for the resulting risk-based decision, and there may be little consistency in the resulting acceptability.
- The third form is to combine the first and second. There is a growing trend within governing bodies to define: the requirement to conduct the analysis, the method to be used, and the acceptable criteria as a set. Organizations such as the Range Commanders Council, the DoD Explosive Safety Board (DDESB) and NATO have adopted such methods in recent years (refs. 6, 7, 8, 9).

The Venerable RAC Code

For the last thirty years or more, System Safety professionals have used the RAC Matrix as a tool in their application of System Safety.

The most commonly used example is from the DOD standard shown in figure 6 (ref. 10). The matrix represents undesired consequence on one axis, and probability on the other. It is very good at conveying the concept that was first described by Pascal; however, there is not agreement as to how well it actually measures risk. The following paragraphs examine the granularity of a generic RAC matrix and focus on its utility as a risk measurement tool.

Mishap Probability Levels	Mishap Severity Categories			
	(1) Catastrophic	(2) Critical	(3) Marginal	(4) Negligible
(A) Frequent	1A	2A	3A	4A
(B) Probable	1B	2B	3B	4B
(C) Occasional	1C	2C	3C	4C
(D) Remote	1D	2D	3D	4D
(E) Improbable	1E	2E	3E	4E

Figure 6 – MIL-STD-882D RAC Matrix

Defining Requirements for a RAC Matrix

Intended Use. As with any tool, the intended use of the matrix is an important requirement. The MIL-STD-882D matrix is sometimes used for qualitative assessments calling for broad non-numerical categories. At other times quantitative risk assessments are used and the same matrix may not apply. Similarly, the same matrix may not apply to risk assessment and risk acceptance when the decision makers are more adverse to catastrophic risk. The intended use is a requirement in designing the proper tool (ref. 11).

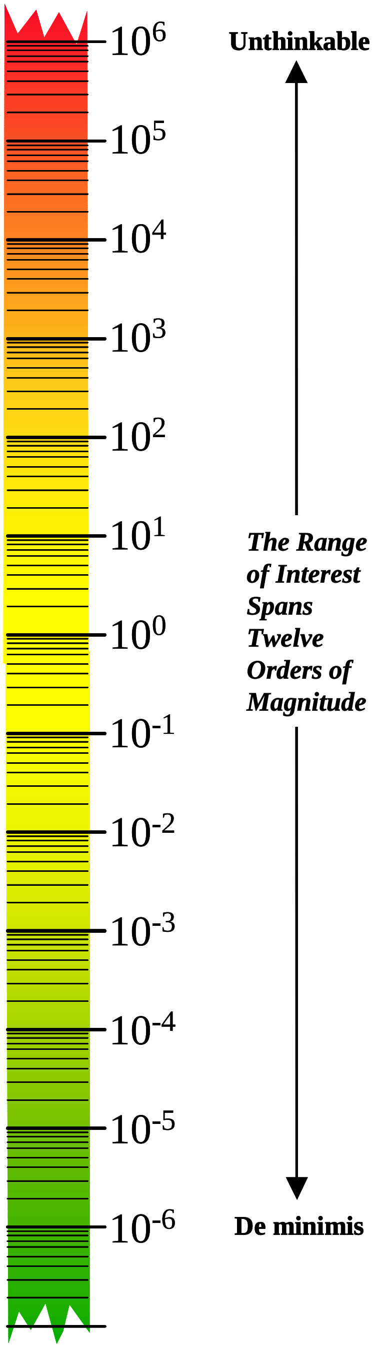


Figure 7 - Risk Scale

Measuring Risk. If our goal is to have a tool to measure risk, and risk is defined as the product of consequence and likelihood, then the definition of the tool to measure the risk can begin by defining the quantitative range of interest for which the tool applies. The generic range of interest for the practicing safety professional has huge variation. At one end of the scale are those events which pose so little risk that they are of no consequence. The legal term *de minimis* applies, which is short for a legal term from Latin "*de minimis non curat lex*," which literally means the law does not concern itself with trifles (ref. 5). This end of the scale is often bounded at the point where the risk of fatality to an individual is no greater than 1 in a million years (ref. 7).

The other end of the scale is more difficult to define clearly. Here the risks are huge, unthinkable. A major war, or large meteor impact, could result in thousands or even millions of fatalities. The point is that the quantitative risk scale varies across such a large span that it is difficult to grasp. A span of 12 orders of magnitude, as shown in figure 7, is not easy to comprehend, especially when our day-to-day frames of reference are almost all presented on a linear scale. Even with the aid of the logarithmic scale it is hard to grasp the difference represented by 12 orders of magnitude.

In a similar fashion each of the two components of risk can be examined. First, consequence: For simplicity we can use a single measure of human injury/fatality as the scale. Here the *de minimis* threshold might be a minor injury requiring no medical aid, and the unthinkable might be thousands of fatalities.

The second component of risk is likelihood, which requires more definition. Likelihood is a measure of probability, and in the context of Pascal's meaning, is for an individual. In a systems context we should expand the term to by multiplying by the number of exposed individuals and exposure time (ref. 12). Because of this multiplication, the range of interest for this term spans a much wider range. At the

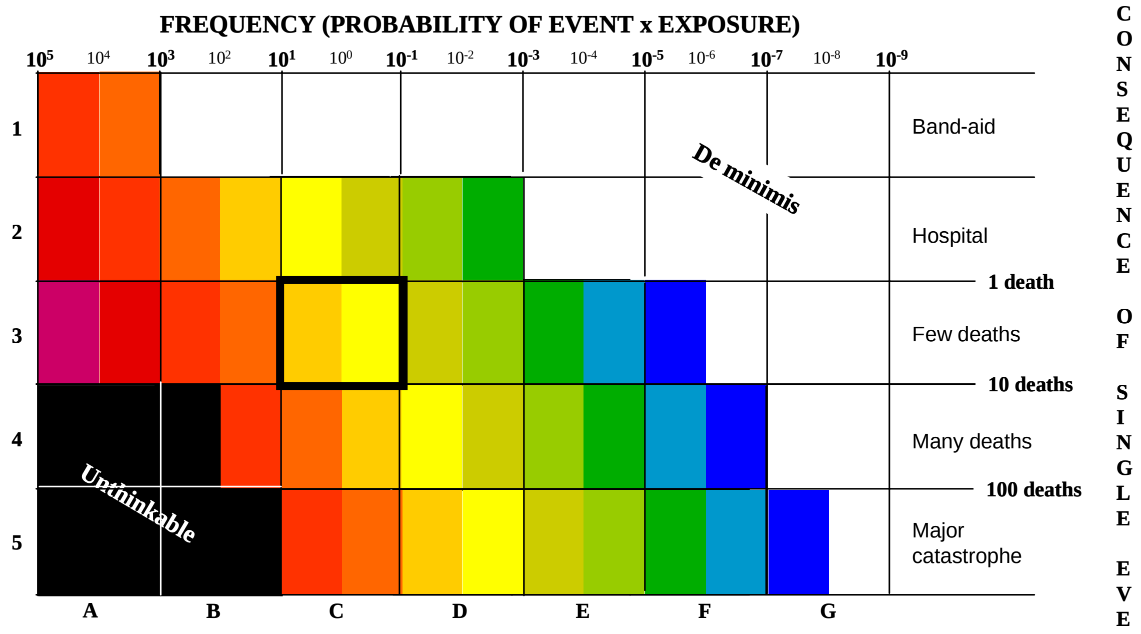


Figure 8 – Generic RAC

upper frequency end, a number such as a thousand times a year for a minor injury is needed, and at the lower frequency end a very small number is needed for the major catastrophic events. As shown in figure 8, 1x10^{-9} might be considered acceptable. This gives a range of interest spanning 14 orders of magnitude!

Figure 8 attempts to combine these two components into a generic RAC matrix that covers the logical range of interest for both factors. The vertical scale, chosen for consequence, is subdivided into five order-of-magnitude increments. The 5 bins are called “Band-Aid,” “Hospital,” “Few deaths,” “Many deaths,” and “Major catastrophe.” The horizontal scale measures frequency and spans a great range. In this RAC, frequency includes not only probability, but also exposure. Seven cells, each with a span of two orders of magnitude are shown in figure 8. This hypothetical RAC matrix uses each color to represent a single risk level. This concept expands Pascal’s formula by including the amount of exposure in the overall risk level. The *de minimis* in the upper right, and unthinkable level lower left are postulated by the authors based on existing industry standards.

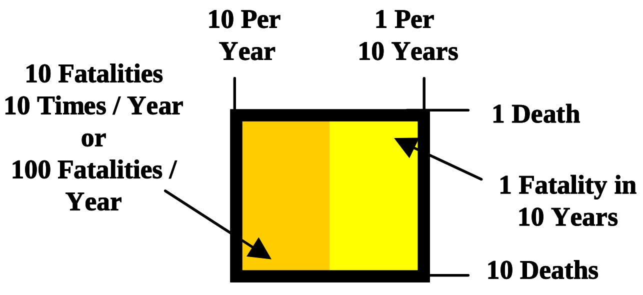
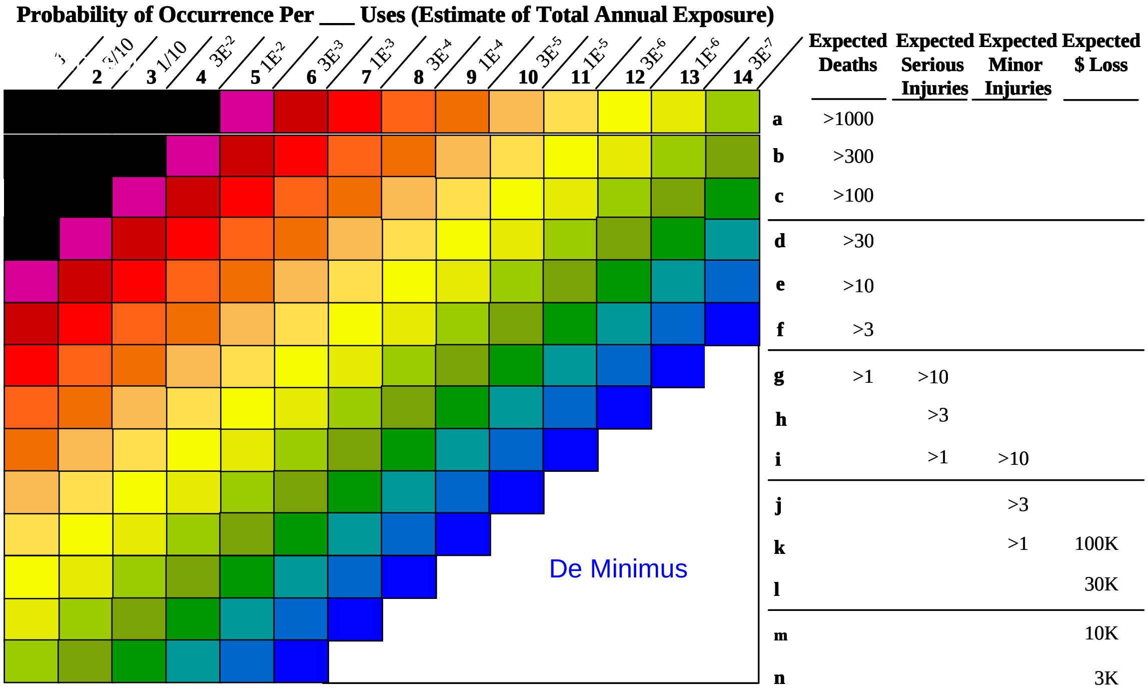


Figure 9 – Resolution of each Cell

When considering the utility of this hypothetical RAC matrix (or any of the other options) as a tool to differentiate risk, the resolution or granularity of the tool is a major factor. For the 5x7 matrix defined above, the resolution is shown by examining the range of risk within a single cell.

Figure 9 illustrates this. As shown, a single cell within the matrix spans one order of magnitude in consequence, two in likelihood, and three in risk. This is a factor of 1,000 within a single cell. It is not very satisfying to attempt to differentiate between optional choices in terms of risk when the tool of choice has a fundamental resolution of a factor of 1000!

There have been attempts to quantify the scales used in the various RACs; however, they are often described as only exemplary. None have developed the underlying mathematics or the



	Expected Deaths	Expected Serious Injuries	Expected Minor Injuries	Expected $ Loss
a	>1000			
b	>300			
c	>100			
d	>30			
e	>10			
f	>3			
g	>1	>10		
h		>3		
i		>1	>10	
j			>3	
k			>1	100K
l				30K
m				10K
n				3K

Figure 10 – Expanded RAC Code

concise definitions needed to build consensus and educate users.

Recent Trends in Governance. In the last few years, several governing bodies have considered the use of expanded RAC matrices for various applications that have more utility as a mathematical tool. One such tool has been proposed to the Insensivitive Munitions community to help assess the benefits of risk reductions due to insensivitive explosives. Figure 10 illustrates the concept, which features granularity, along each axis, of ½ order of magnitude. A similar concept removes all granularity by using continuous scales along each axis, resulting in infinite resolution.

The examples above serve to illustrate the difficulty of having a quantified generic tool. Because of the granularity issue and the huge span of interest, it is not easy to conceive of a single tool that could gain acceptance among the many applications. Nevertheless, by using concise definitions and protocols like those postulated by Pascal, a self-consistent and universal tool could be defined.

Conclusion

The major conclusion of this paper is this:

The concept originated by Pascal was clearly mathematical in nature. Its application to date in the form of a RAC matrix is, however, only accurate at the conceptual level. We need to move forward to define better tools.

Pascal, the logical thinker, pointed us to an answer: “Fear of harm ought be proportional to the gravity of the harm and the likelihood of the event.” This simple statement clearly points us toward a concept. We have captured the concept in our RAC matrix. Pascal, however, was primarily a mathematician. His words state a clear mathematical equation that the RAC matrix has only begun to capture adequately.

The challenge therefore, posed by this paper to this conference some 339 years after Pascal showed us the way ahead, is to follow his uncomplicated direction to its logical conclusion. We cannot claim that System Safety Engineering is a separate discipline when we have only refined our tools to a conceptual level. We must *add discipline* to our chosen discipline. We should encourage, sponsor, endorse, develop and build consensus for methods that apply more mathematical discipline, rigor and consistency.

Recommendation

The authors recommend that a quantitative RAC (or series of alternative RACs) be defined which apply the uncomplicated mathematics postulated by Pascal. For each RAC we should define its intended use as a standard for risk assessment, or risk acceptance, or both. As a set, they should be applicable across the broad set of circumstances encountered by the practicing safety professional.

References

1. Arnauld, Antoine and Nicole, Pierre. Logic or the Art of Thinking. 1996.
2. Bernstein, Peter L. Against the Gods, The Remarkable Story of Risk. 1998.
3. Hacking, Ian. The Emergence of Probability. 1975.
4. Muir, Jane. Of Men and Numbers, The Story of the Great Mathematicians. 1996.
5. Standard 321-00. "Common Risk Criteria for National Test Ranges, Subtitle: Inert Debris." April 2000. published by Secretariat, Range Commanders Council, U.S. Army White Sands Missile Range, New Mexico 88002-5110.
6. DDESB Technical Paper #14, "Explosives Safety Risk Analysis." February 2000. http://www.hqda.army.mil/ddesb/documents.html.
7. Tom Pfitzer, Meredith Hardwick, Paul Price, and Jerry Ward. "Status of the Risk-Based Explosives Safety Criteria Team." Proceedings from the 29th DDESB Explosives Safety Seminar. New Orleans, Louisiana, 18-20 July 2000.
8. NATO AC/258 WP 212, AASTP-4, “Explosives Safety Risk Analysis.” February 2001.
9. Tom Pfitzer, Jerry Rufe, and Jerry Ward. "Criteria Selection for Risk-Based Explosives Safety Standards." Proceedings from the Parari '99 Seminar. Canberra, Australia, 10-12 November 1999.
10. MIL-STD-882D. Standard Practice for System Safety. 2000.
11. P.L. Clemens. “Preferences in Interpreting the Risk Assessment Matrix.” ASSE Journal, June 1995.
12. P.L. Clemens and R.J. Simmons. “The Risk Exposure Interval – Too Often an Analyst’s Trap.” *Journal of System Safety*, 1st Quarter, 2001.

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He is currently the leader of teams supported by two U.S. Government agencies that are in the process of promulgating new risk-based standards. The National Range Commander's Council (RCC) recently published a risk-based standard for debris protection that was prepared by a team he led. He was also a leader of the government/contractor team that developed and promulgated a risk-based standard for the DoD Explosive Safety Board. This board sets national policy for explosives safety. He serves as the US member to the expert-working group that recently published the NATO risk-based standard. His management efforts to expand System Safety concepts and methods into other safety disciplines resulted in the 1999 Manager of the Year Award from the International System Safety Society.

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Ms. Dwyer is active in the System Safety Society, has served multiple terms as an officer in the local chapter, and was recognized as the local chapter Professional of the Year in 1998.